Incomplete noise-induced synchronization of spatially extended systems

Alexander E. Hramov, Alexey A. Koronovskii, and Pavel V. Popov
Faculty of Nonlinear Processes, Saratov State University, Astrakhanskaya, 83, Saratov, 410012, Russia
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A type of noise-induced synchronous behavior is described. This phenomenon, called incomplete noise-induced synchronization, arises for one-dimensional Ginzburg-Landau equations driven by common noise. The mechanisms resulting in incomplete noise-induced synchronization in spatially extended systems are revealed analytically. Different types of model noise are considered. A very good agreement between the theoretical results and the numerically calculated data is shown.

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I. INTRODUCTION

Noise-induced synchonization [1–3] is a ubiquitous phenomenon in nonlinear science. It arises in the interplay between determined and random dynamics [4], with both the synchronization and noise influence being recently subjects of considerable interest in the scientific community. Indeed, on the one hand the synchronous behavior of nonlinear systems has attracted great attention of researchers for a long time [5–8]. On the other hand, discovering the fact that fluctuations can actually induce some degree of order in a large variety of nonlinear systems is one of the most surprising results of the last decades in the field of stochastic processes [9–11]. Moreover, both these phenomena are relevant for physical, chemical, biological, and other systems described in terms of nonlinear dynamics (see, e.g., [12–15]).

Noise-induced synchronization (NIS) means that the random signal influencing two identical uncoupled dynamical chaotic systems $\mathbf{u}(t)$ and $\mathbf{v}(t)$ [starting from the different initial conditions $\mathbf{u}(t_0)$ and $\mathbf{v}(t_0)$, $\mathbf{u}(t_0)\neq\mathbf{v}(t_0)$] results in their synchronous (i.e., identical) behavior $\mathbf{u}(t)=\mathbf{v}(t)$ after a transient.

Noise-induced synchronization can be detected by means of direct comparison of the states of two chaotic systems $\mathbf{u}(t)$ and $\mathbf{v}(t)$ under the influence of noise. The other method of diagnostics of NIS is calculation of the largest Lyapounov exponent (LE) of a dynamical system, which measures the stability of the motion. Indeed, in driven chaotic systems the largest Lyapunov exponent may become negative, which results in synchronization: two systems forced by the same noise "forget" their initial conditions and evolve to an identical state [16]. If the noise influence is infinitely small the largest Lyapunov exponent is positive for such a system.

In all cases of noise-induced synchronization known hitherto, the boundary of the noise-induced synchronization regime is associated with the point on the parameter axis where the largest Lyapunov exponent of the system under study crosses the zero value and its sign is changed from "plus" to "minus." In this paper we report that the noise-induced synchronization regime of two spatially extended uncoupled identical systems driven by common noise may be preceded by a different type of behavior, when the largest Lyapunov exponent remains zero in a finite range of parameter values. This kind of behavior called *incomplete noise-induced synchronization* (INIS) demonstrates the features of the synchronous motion of two uncoupled identical systems driven by common noise: although the states of the system differ, trajectories can be transformed into each other by an appropriate spatial shift.

The structure of the paper is the following. In Sec. I we describe the system under study: the complex Ginzburg-Landau equations driven by common noise and their behavior (incomplete noise-induced synchronization) observed for a certain set of values of the control parameters. In Sec. II we consider the mechanisms that are responsible for the INIS regime. Section III deals with the different models of noise with distinct probability densities. The final conclusions are given in Sec. IV.

II. SYSTEM UNDER STUDY AND INCOMPLETE NOISE-INDUCED SYNCHRONIZATION

The system under study is represented by a pair of uncoupled complex Ginzburg-Landau equations (CGLEs) driven by common noise:

$$u_t = u-(1-i\beta)|u|^2u+(1+i\alpha)u_{xx}+D\zeta(x,t),$$

$$v_t = v-(1-i\beta)|v|^2v+(1+i\alpha)v_{xx}+D\zeta(x,t), \qquad (1)$$

where $u(x,t)$ and $v(x,t)$ are complex states of the considered systems, α and $\beta=4$ are the control parameters, and D defines the intensity of the noise term $\zeta=\zeta_1+i\zeta_2$. We have used model noise with an asymmetrical probability distribution of the real ζ_1 and imaginary ζ_2 parts of the random variable,

$$p(\zeta_{1,2})=\begin{cases}2\zeta_{1,2} & \text{if } 0\leq\zeta_{1,2}\leq 1,\\ 0 & \text{otherwise,}\end{cases} \qquad (2)$$

on the unit interval $\zeta_{1,2}\in[0;1]$. The simulation of the random variables $\zeta_{1,2}$ with required probability distribution $p(\zeta_{1,2})$ was carried out in the same way as described in [17] for the exponential staggered distribution. Equation (1) was solved with periodic boundary conditions

$$u(x,t)=u(x+L,t) \quad \text{and} \quad v(x,t)=v(x+L,t), \qquad (3)$$

with all numerical calculations being performed for a fixed system length $L=40\pi$ and random initial conditions. To evaluate (1) the standard numerical scheme was used [18]; the value of the grid spacing is $\Delta x=L/1024$, and the time step of the scheme $\Delta t=2.0\times10^{-4}$.

If the noise intensity is equal to zero ($D=0$) and the initial conditions $u(x,0)$ and $v(x,0)$ are not identical, both systems demonstrate complex chaotic behavior (both in time and in

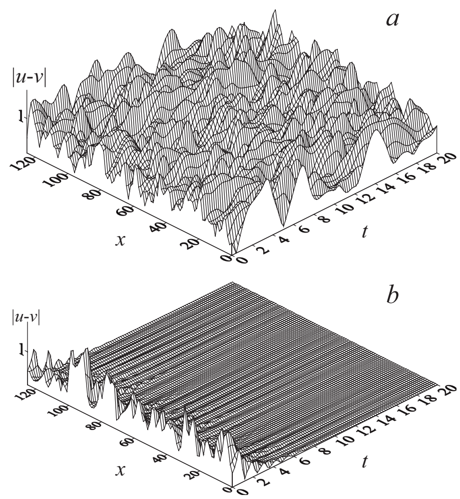

FIG. 1. Evolution of the difference of the system states $|u(x,t)-v(x,t)|$ described by complex Ginzburg-Landau equations (1) (a) without noise and (b) with noise with intensity $D=3$. In the second case the difference of the states of both systems at every point of space tends to zero (after a transient), which indicates the presence of the noise-induced synchronization regime. The control parameter values are $\alpha=2$ and $\beta=4$.

space), with the system states being different, i.e., $u(x,t)\neq v(x,t)$ [Fig. 1(a)]. Alternatively, if the noise intensity D is large enough the states of both systems coincide with each other [Fig. 1(b)], which is evidence of noise-induced synchronization.

To detect the presence of the noise-induced synchronization regime the averaged difference

$$\varepsilon=\frac{1}{TL}\int_{\tau}^{\tau+T}\int_{0}^{L}|u(x,t)-v(x,t)|dx\,dt, \qquad (4)$$

between the spatiotemporal states of two CGLEs driven by common noise was calculated. The averaging process starts after a long-time transient with duration $\tau=200$.

In the NIS regime we have the relation $\varepsilon=0$, since in this case the difference between the states of two identical spatially extended systems (1) at every point of space tends to zero. We have also calculated the largest Lyapunov exponent λ for one of the systems (1). As mentioned above, in the NIS regime the largest Lyapunov exponent λ should be negative.

The dependencies of the largest Lyapunov exponent $\lambda(D)$ and the averaged difference $\varepsilon(D)$ on the noise intensity D are shown in Fig. 2 for two different values of the control parameter α. It is easy to see that for the control parameter $\alpha=1$ [curves 1 in Figs. 2(a) and 2(b)] the value of the noise intensity D for which the largest Lyapunov exponent λ crosses zero and becomes negative coincides with the point where the averaged difference (4) starts being vanishingly small. So in this case the noise-induced synchronization boundary is $D_{\mathrm{NIS}}\approx 1.5$ [see arrows in Figs. 2(a) and 2(b)], and we deal with the occurrence of the typical and well-known noise-induced synchronization regime.

Alternatively, a different scenario is observed in the same system (1) if the control parameter value $\alpha=2$ is considered

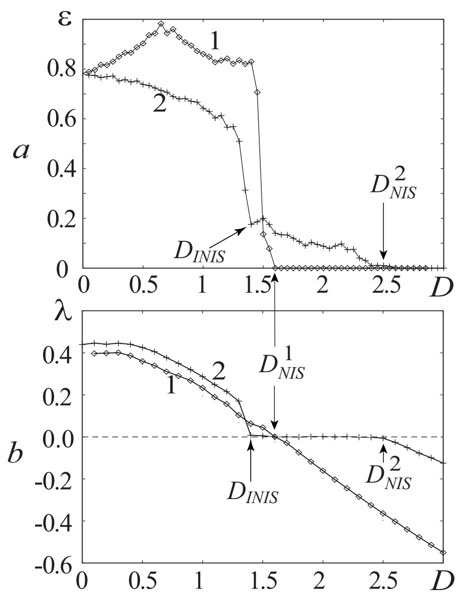

FIG. 2. Dependencies of (a) the averaged difference (4) and (b) the largest Lyapunov exponent of the CGLE on the noise intensity D for different values of the control parameter α. Curves 1 correspond to the case of $\alpha=1$; curves 2 were calculated for $\alpha=2$. The values of noise intensity corresponding to the onset of noise-induced synchronization are shown by arrows with labels D^1_{NIS} and D^2_{NIS} for the curves 1 and 2, respectively. The boundary of the incomplete noise-induced synchronization is also shown by the arrow marked D_{INIS}.

[see curves 2 in Figs. 2(a) and 2(b)]. For such a choice of α-parameter value the largest Lyapunov exponent becomes equal to zero for a large enough intensity of noise $D_{\mathrm{INIS}}\approx 1.53$, whereas the averaged difference ε between the spatiotemporal states of two CGLEs driven by common noise exceeds zero [Figs. 2(a) and 2(b)]. With further increase of the noise intensity D (when D is equal to $D_{\mathrm{NIS}}\approx 2.5$) the value of ε becomes equal to zero [see Fig. 2(a)] and the largest Lyapunov exponent starts to be negative, which is the evidence of the presence of a noise-induced synchronization regime.

In other words, there is a finite interval of the noise intensity values $(D_{\mathrm{INIS}}, D_{\mathrm{NIS}})$ for which the noise-induced synchronization is not observed, and the largest Lyapunov exponent λ is equal to zero. To prove this fact we have calculated the largest Lyapunov exponent of the complex Ginzburg-Landau equation for different values of the spatial grid spacing. The range of noise intensities corresponding to the plateau where $\lambda=0$ is shown in Fig. 3. One can see that the largest Lyapunov exponent calculations with different values of the spatial grid step give similar results. Based on these calculations, we come to the conclusion that the largest Lyapunov exponent is actually equal to zero in a finite range of the noise intensity.

Despite the fact that the noise-induced synchronization is not observed in the region where $\lambda=0$ (see Fig. 2), this range of noise intensities corresponds to behavior showing the features of synchronous dynamics. The manifestation of synchronism can be observed if one of the complex media described by the Ginzburg-Landau equation starts to shift

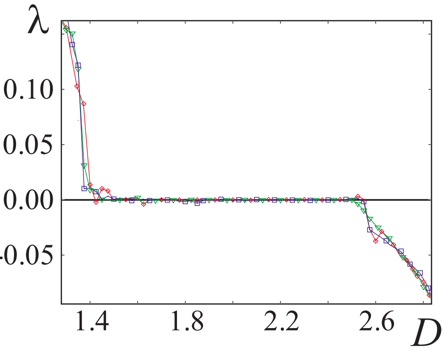

FIG. 3. (Color online) Dependencies of the largest Lyapunov exponent λ on the noise intensity D calculated for different values of the spatial grid spacing: □, $\Delta x = L/2^{10}$, $\Delta t = 2.0\times10^{-4}$; ◇, $\Delta x = L/2^{11}$, $\Delta t = 1.0\times10^{-4}$; ▽, $\Delta x = L/2^{12}$, $\Delta t = 5.0\times10^{-5}$. The plateau where $\lambda = 0$ is shown. The control parameter values are $\alpha = 2$ and $\beta = 4$.

compared to the second one with spatial shift δ. In other words, if one uses the shifted state of one of the systems, $v = v(x+\delta, t)$, in Eq. (1), the averaged difference ε changes depending on this shift δ. This movement of one of the systems must be very slow for the transient to be completed. In this case such a spatial shift δ_0 may be found that both Ginzburg-Landau equations start to behave identically, with the largest Lyapunov exponent being equal to zero. Therefore, we have called this regime incomplete noise-induced synchronization.

This statement is illustrated in Fig. 4 where the dependence of the difference ε (4) on the space shift δ is shown. One can see that there is a value δ_0 of the shift δ for which the averaged difference ε becomes equal to zero. Therefore, for this space shift δ_0 both systems demonstrate identical behavior and the noise-induced synchronization is observed. This shift δ_0 depends on the initial conditions. For the other values of the spatial shift δ the system states (in both space and time) are different, but the largest Lyapunov exponent is

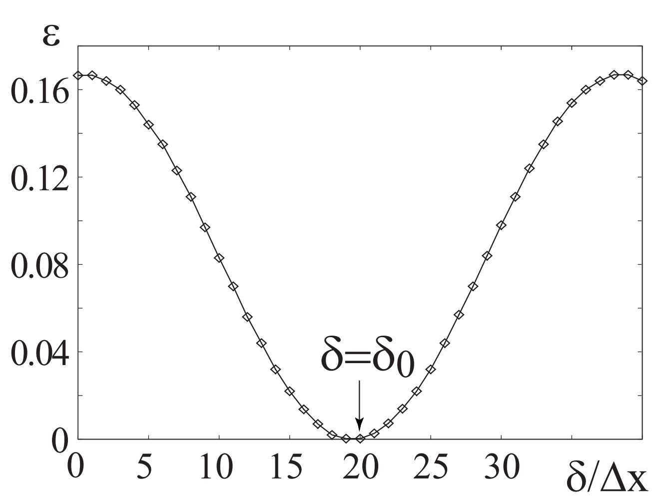

FIG. 4. Dependence of the difference ε between the states of the media $u(x,t)$ and $v(x,t)$ described by the complex Ginzburg-Landau equations (1) on the space shift δ for the control parameters $\alpha = 2$, $\beta = 4$, and $D = 2$.

always equal to zero for the considered set of control parameter values.

III. MECHANISMS RESULTING IN THE INCOMPLETE NOISE-INDUCED SYNCHRONIZATION REGIME

Let us discuss the mechanisms resulting in the occurrence of the incomplete noise-induced synchronization regime. In [4] it has been shown that, for dynamical systems with a small number of degrees of freedom, the mechanisms of the onset of noise-induced synchronization and generalized synchronization are equivalent. The mechanism of generalized synchronization can be considered with the help of the modified system approach as in Ref. [19] for chaotic systems with a small number of degrees of freedom and in Ref. [20] for a spatially extended system. We interpret the mechanism of the onset of incomplete noise-induced synchronization in a similar way. Therefore, following Refs. [4,19,20], we consider the dynamics of a modified spatially extended system with the additional term determined by the mean value of the noise.

The deterministic modified Ginzburg-Landau equation with the additional term, determined by the mean value $\langle D\zeta\rangle$ of the noise term ζ in the stochastic equation (1) can be written as

$$\frac{\partial u_m}{\partial t} = u_m - (1 - i\beta)|u_m|^2 u_m + (1 + i\alpha)\frac{\partial^2 u_m}{\partial x^2} + \langle D\zeta\rangle. \quad (5)$$

For the selected kind of noise with the probability distribution (2) $\langle D\zeta\rangle = 2D/3$.

Equation (5) is a forced CGLE, widely studied and well documented in the literature (see, e.g., [21–23]). It is well known that different types of spatiotemporal patterns may be observed depending on the domain of the control parameter values. If the value D is large enough, the homogeneous stationary state $u_0 = u_0(x,t) = \text{const}$ is observed in the system (5). In this case the largest Lyapunov exponent is negative, with the stationary state regime in the system (5) corresponding to the noise-induced synchronization in the system (1). With decrease of the noise intensity D the stationary state u_0 loses its stability, which corresponds to the boundary of the noise-induced synchronization of the initial Ginsburg-Landau equations (1) driven by noise.

At the same time the loss of the stability of the homogeneous stationary state occurs in different ways depending on the control parameter values of the modified Ginzburg-Landau equation (5).

Indeed, the homogeneous stationary state u_0 can be obtained numerically from the equation

$$u_0 - (1 - i\beta)|u_0|^2 u_0 + 2D/3 = 0, \quad (6)$$

e.g., by the Newton method [24]. To analyze the stability of Eq. (6) we have to consider the linearization of the modified Ginzburg-Landau equation in the vicinity of the stationary solution u_0. Let $\tilde{u} = \tilde{u}_r + i\tilde{u}_i$ be a small perturbation of the homogeneous stationary state $u_0 = u_r + iu_i$, i.e., $u_m = u_0 + \tilde{u}$. Having linearized equation (5) and assuming that $\tilde{u}_r(x,t) = \hat{u}_r(k)\exp(\Lambda t + ikx)$, $\tilde{u}_i(x,t) = \hat{u}_i(k)\exp(\Lambda t + ikx)$, we obtain the dispersion relation

$$\begin{vmatrix} 1-u_i^2-3u_r^2-2\beta u_i u_r-k^2-\Lambda & -(\beta u_r^2+3\beta u_i^2+2u_r u_i-\alpha k^2) \\ \beta u_i^2-2u_i u_r+3\beta u_r^2-\alpha k^2 & 2\beta u_r u_i-u_r^2-3u_i^2+1-k^2-\Lambda \end{vmatrix}=0 \quad (7)$$

determining the stability of the homogeneous stationary state u_0. The homogenous stationary state u_0 is stable if the condition Re $\Lambda(k)<0$, $\forall\ k$, is satisfied.

The evolution of Re $\Lambda(k)$ with the decrease of D for $\alpha=1$ and 2 is shown in Figs. 5(a) 5(b), respectively. One can see that for $\alpha=1$ the homogeneous stationary state u_0 loses its stability when $D\approx 1.5$. In this case the spatial perturbation with the wave number $k=0$ starts growing exponentially. As a result, the stationary state u_0 becomes unstable, spatiotemporal chaos occurring in the system (5). With the largest Lyapunov exponent becoming positive in both the modified (5) and original (1) Ginzburg-Landau equations, the noise-induced synchronization regime in Eq. (1) is destroyed.

For the value of the control parameter $\alpha=2$ the homogeneous stationary state u_0 loses its stability for $D\approx 2.5$ and the spatial mode with the wave number $k=\pm 0.5$ becomes unstable, in contrast to the case of $\alpha=1$ considered before [see Fig. 5(b)]. Therefore, for $\alpha=2$ the periodic spatial state $u_k(x)=u_k(x+l)$ (where l is close to $2\pi/k$ due to the periodic boundary conditions), which is stationary in time, replaces the homogeneous state u_0 in the modified Ginzburg-Landau equation. An example of the profiles of such stationary-in-time but periodic-in-space states is shown in Fig. 6. Obviously, for such stationary states the largest Lyapunov exponent is equal to zero. Evidently, in the initial Ginzburg-Landau equation driven by noise $D\zeta(x,t)$ with the mean value $\langle D\zeta\rangle$, the structure $u_k(x)$, stationary in time and periodic in space, is perturbed by fluctuations. Therefore, the spatiotemporal dynamics of $u_k(x)$ looks like aperiodic motion, with the largest Lyapunov exponent being also equal to zero. Since the two identical media $u(x,t)$ and $v(x,t)$, driven by common noise, start with different initial conditions $u(x,0)$ and $v(x,0)$, the spatially periodic structures do not coincide with each other, i.e., $u_k(x)\neq v_k(x)$, but there is a shift in space δ_0 depending on the initial conditions $u(x,0)$ and $v(x,0)$ where $u_k(x)=v_k(x+\delta_0)$. Therefore, for $D_{\text{INIS}}<D<D_{\text{NIS}}$ the Ginzburg-Landau equations (1) driven by common noise are characterized by zero largest Lyapunov exponent, and their states are not identical. If the first of the systems is shifted compared to the second one with a certain shift δ_0 that, depending on initial conditions, satisfies the requirement $u_k(x)=v_k(x+\delta_0)$, identical behavior of both systems is observed.

Note also the very good agreement between the values of the noise intensity D corresponding to the loss of stability of the homogeneous stationary state u_0 [see Fig. 5(b)] and to the point where the largest Lyapunov exponent becomes equal to zero [see Fig. 2(b)].

It should be noted that the INIS phenomenon considered above is determined by the peculiarity of the periodic boundary conditions. Because of the use of this kind of boundary conditions, any spatially periodic solution can be moved arbitrarily along the spatial coordinate, and, therefore, in this case there is an additional translational degree of freedom. Clearly, the zero Lyapunov exponent corresponds to this translational degree of freedom, whereas all other Lyapunov exponents are negative. Instantaneous states evolved from different initial conditions under the influence of the given realization of noise can be transformed into each other by means of the corresponding spatial shift. Taking this aspect into consideration, the INIS regime does not seem to be ob-

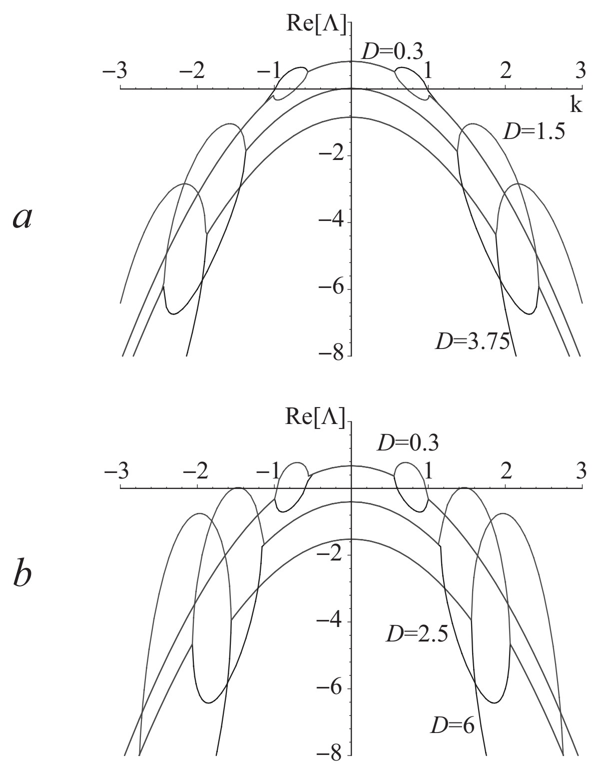

FIG. 5. Dependencies of the real part of the eigenvalues Λ on the wave number k for different values of the D parameter when the control parameter α has been fixed as $\alpha=$ (a) 1 and (b) 2.

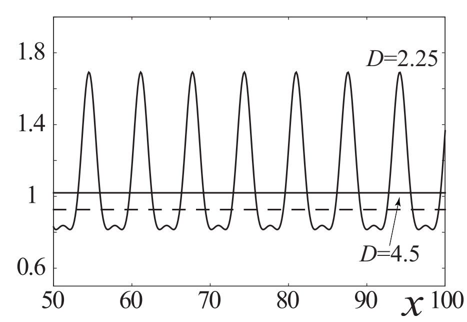

FIG. 6. Profiles of the spatial stationary states $|u_0|^2$ and $|u_k(x)|^2$ observed in (5) for different values of the noise intensity D and $\alpha=2$. Dashed lines correspond to the unstable state $|z_0|^2$.

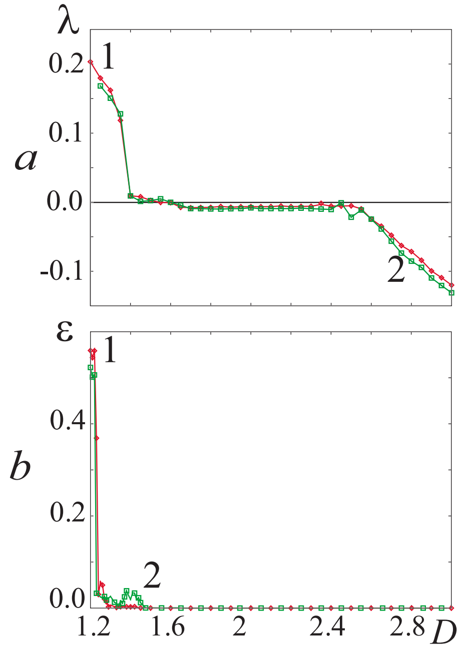

FIG. 7. (Color online) Dependencies of (a) the largest Lyapunov exponent of the CGLE and (b) the averaged difference (4) on the noise intensity D for control parameter values $\alpha=2$ and $\beta=4$. Curves 1 (♦) correspond to the boundary conditions (8) and curves 2 (□) to the boundary conditions (9).

served in the absence of translational invariance, i.e., for other kinds of boundary conditions.

To illustrate this statement, we have considered the complex dynamics of the Ginsburg-Landau equation (1) with the same set of the control parameter values but with alternative types of boundary conditions, eliminating the translational invariance in the system under study. We have calculated both the largest Lyapunov exponent λ and the averaged difference ε for two types of boundary conditions:

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0,$$

$$\left.\frac{\partial v}{\partial x}\right|_{x=0} = \left.\frac{\partial v}{\partial x}\right|_{x=L} = 0, \tag{8}$$

and

$$\mathrm{Re}\, u(0,t) = \mathrm{Re}\, u(L,t) = 1,$$

$$\mathrm{Im}\, u(0,t) = \mathrm{Im}\, u(L,t) = 1,$$

$$\mathrm{Re}\, v(0,t) = \mathrm{Re}\, v(L,t) = 1,$$

$$\mathrm{Im}\, v(0,t) = \mathrm{Im}\, v(L,t) = 1. \tag{9}$$

The obtained curves $\lambda(D)$ and $\varepsilon(D)$ are shown in Fig. 7.

Clearly, in both cases there is no translational invariance in the system, and, as a result, the largest Lyapunov exponent is close to zero, but negative, in the range of D values where for the periodic boundary conditions (3) the INIS regime is observed. The averaged difference ε, in turn, is equal to zero

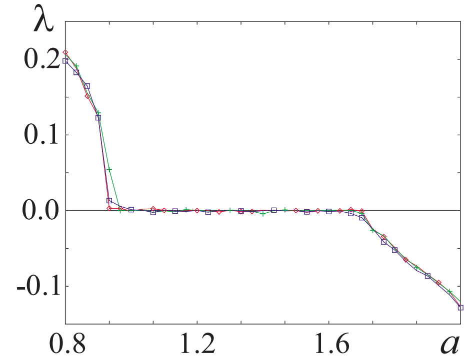

FIG. 8. (Color online) Dependencies of the largest Lyapunov exponent λ on the value a calculated for noise with Gaussian distribution (10) for different values of the variance: ◇, $\sigma=0.1$; +, $\sigma=0.2$; □, $\sigma=0.5$. A plateau where $\lambda=0$ is observed. The control parameter values of the complex Ginzburg-Landau equation are $\alpha=2$ and $\beta=4$.

in this range of noise intensity values, which shows the impossibility of existence of the INIS regime if there is no translational invariance in the system.

IV. INCOMPLETE NOISE-INDUCED SYNCHRONIZATION AND THE NOISE CHARACTERISTICS

To illustrate that the onset of the incomplete noise-induced synchronization regime is caused by the mean value of noise only, we examine how the different types of model noise influence the considered media described by CGLEs (1). One of the typical probability densities is the Gaussian; therefore, it seems reasonable to consider the dynamics of the complex Ginzburg-Landau equations (1) driven by common noise with a Gaussian probability distribution of the real and imaginary parts of the random variable,

$$p(\xi) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\xi-a)^2}{2\sigma^2}\right). \tag{10}$$

Although in Secs. I and II we were not able to separate the influence of the mean value and variance from each other, we can do it easily for Gaussian probability density. To do that, the value of the noise intensity $D=1$ has been fixed. In this case the mean value of the random process is governed by the choice of a parameter, while the intensity of noise is determined by the variance σ. The random variable with the required probability density has been generated following Ref. [24].

We have calculated the dependence of the largest Lyapunov exponent λ on the mean value a of the probability density (10) for different values of the variance σ. The results of these calculations are given in Fig. 8 where the curves λ vs a are shown. As follows from the numerical calculations, the incomplete noise-induced synchronization is observed in the range $a\in[1.0,1.7]$ (as well as in the case considered in Sec. I) for all values of the variance σ. Therefore, the value of the variance σ (i.e., the noise intensity) does not seem to play a key role in the occurrence of the INIS regime.

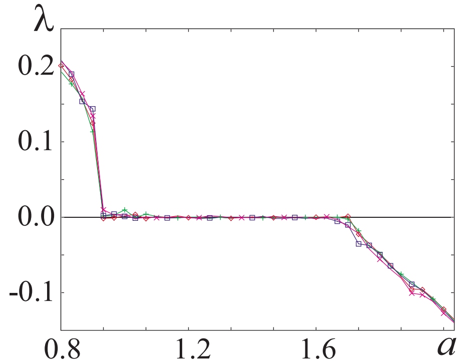

FIG. 9. (Color online) Dependencies of the largest Lyapunov exponent λ on the value a calculated for a uniform distribution of the random variable with different values of the variance: ◇, $\sigma=0.5$; +, $\sigma=1.0$; □, $\sigma=1.5$; ×, $\sigma=2.0$. A plateau where $\lambda=0$ is observed. The control parameter values of the complex Ginzburg-Landau equation are $\alpha=2$ and $\beta=4$.

Having examined the spatiotemporal behavior of two complex Ginzburg-Landau equations (1) driven by common noise with the probability density (10), we have found for the range $a\in[1.0,1.7]$ that their states do not coincide with each other; therefore there is no noise-induced synchronization as expected. Nevertheless, there is a spatial shift δ for which the dynamics of systems $u(x,t)$ and $v(x+\delta,t)$ are identical, which is also evidence of the presence of the incomplete noise-induced synchronization regime.

To compare the results described in Secs. II and III with each other we have to take into account that the value $2D/3$ has been substituted in Sec. II for the mean value of noise [see the explanation given below Eq. (5)]. Obviously, if we substitute the same value $2D/3$ for the mean value a (i.e., denote $a=2D/3$) in the case of noise with a Gaussian probability distribution, we obtain that the range of the INIS regime is $D\in[1.5,2.5]$, which agrees very well with the results of the analytical consideration given in Sec. II.

The very same results have been obtained for a uniform distribution of random variables with the mean value a and variance σ (Fig. 9): there is a range of values of the a parameter where the largest Lyapunov exponent is equal to zero and the incomplete noise-induced synchronization regime occurs.

Based on this consideration, we come to the conclusion that the occurrence of the incomplete noise-induced synchronization regime is determined by the mean value of the noise, whereas its variation plays practically no role.

V. CONCLUSION

In conclusion, we have reported a type of noise-induced synchronous behavior occurring in spatially extended systems. This type of incomplete noise-induced synchronization differs remarkably from other types of synchronous behavior known so far. It may be observed in a certain range of the noise intensity values, where the largest Lyapunov exponent is equal to zero and the states of two identical spatially extended systems driven by common noise are different, although there is an indication of synchronism: if one of the systems is shifted compared to the second one at a certain shift identical behavior of the considered systems is observed. The theoretical equations allowing us to explain the mechanism resulting in this type of behavior have also been given, and they are in perfect agreement with the numerically obtained data. The influence of a different model noise on the occurrence of the noise-induced synchronization regime was considered. Although the INIS regime has been observed here in complex Ginzburg-Landau equations driven by common noise with nonzero mean value, we expect that the very same type of behavior can be observed in many other relevant circumstances. Since the noise influence may result in pattern formation (see, e.g., [25]), we suppose that incomplete noise-induced synchronization can also be observed for noise with zero mean value, with other types of spatiotemporal patterns (e.g., traveling waves) being observed.

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